



Munich Personal RePEc Archive

The Transformation Problem: A Tale of Two Interpretations

David Bieri

Virginia Tech

15. May 2007

Online at <http://mpra.ub.uni-muenchen.de/3895/>

MPRA Paper No. 3895, posted 6. July 2007

The Transformation Problem: A Tale of Two Interpretations *

David S. Bieri[†]

First version: April 2007

This version: May 30, 2007.

Abstract

Over 100 years since Marx's value theory of labour was first published, the so-called "transformation problem" – deriving prices from values and providing a theory of profits as arising from surplus value – has inspired the imagination of economist of all shades of intellectual suasion. However, while mainstream economists have by and large come to dismiss the transformation problem as a trivial technical exercise, the issue has recently received renewed attention in Marxian economic theory. This paper provides a broad historical overview of the transformation problem and specifically focuses on similarities and differences of how the transformation problem has been interpreted, why it was put to rest in mainstream economics and how it has regained prominence in Marxian economics.

JEL Classification: B1, B2, B41

Key Words: History of Economic Thought, Marxian Economics, Value Theory of Labour.

*I would like to thank Hans Haller for helpful comments and suggestions.

[†]David Bieri is a Ph.D. student at the School of Public & International Affairs at Virginia Tech, Blacksburg, VA 24060, USA. E-mail: dsb@vt.edu.

Contents

I	Introduction	1
II	Marx’s Labour Theory of Value	2
II.1	Labour as a Numéraire	2
II.2	From Values to Prices	3
II.3	The Marxian Input-Output System	4
III	The Transformation Problem	5
III.1	Solving the Transformation Problem	7
III.2	Böhm-Bawerk and Bortkiewicz	8
III.3	Beyond Mathematics	11
IV	Two Interpretations	11
IV.1	Transforming the Mainstream?	12
IV.2	The “New Solution”	14
V	Outlook	15

I Introduction

Over 100 years since Marx’s value theory of labour was first published, the so-called “transformation problem” – deriving prices from values and providing a theory of profits as arising from surplus value – has inspired the imagination of economist of all shades of intellectual suasion. Although it is generally accepted that the transformation problem has been solved (at least mathematically) in a number of ways, there are few problems in economics that have excited similar interest over such a prolonged period. Indeed, (Skousen, 2007, p.93) refers to the transformation problem as the “most glaring single hole in the Marxian model”.

The transformation problem may have “been solved and resolved in a number of ways”¹, yet economists’ fascination with this topic seems almost strong as it was over a century ago. After a several waves of increasing and decreasing attention – the most famous waves perhaps having been triggered by the writings of von Böhm-Bawerk (1898), Winternitz (1948) and then by Samuelson (1971) – there has been a flurry of renewed interest in the topic, primarily spurred by a so-called “new solution” to the transformation problem in recent years.

However, while mainstream economists have by and large come to dismiss the transformation problem as a trivial technical exercise, the issue has recently received renewed attention in Marxian economic theory. This paper provides a broad historical overview of the transformation problem and specifically focuses on similarities and differences of how the transformation problem has been interpreted, why it was put to rest in mainstream economics and how it has regained prominence in Marxian economics.

The remainder of this paper is structured as follows. Section II provides an overview of Marx’s labour theory of value and section III describes the transformation problem and illustrates the main solutions that have emerged dealing with this issue. Section IV then puts two contrasting interpretations of these solutions into a historical context. Lastly, section V offers some concluding remarks.

¹Desai (1988, p.297).

II Marx's Labour Theory of Value

Labour theory of value (LTV) is a central feature of classical economics and proportionally relates the exchange value of a commodity to the amount of labour that was required to produce it. Although LTV is perhaps most widely associated with Marxian economics today, its theoretical foundations were laid by Adam Smith and David Ricardo and were adopted Marx without much modification. Indeed, LTV was the predominant theoretical paradigm regarding the determinants of economic value right until the mid-nineteenth century when it was increasingly replaced by theories of marginal utility.

II.1 Labour as a Numéraire

LTV is a catch-all term that encompasses several all of which share the fact that labour serves as a measure of exchange value. Smith's beaver-deer example is perhaps the most well-known illustration of how labour is used as a numéraire. While this is certainly not the place to provide a complete review of different varieties of LTV, it may still be useful to highlight some of the most important elements.

Labour demand theory suggest that the exchange value of a commodity is equal to the quantity of labour that can be demanded in exchange. Similarly, *labour cost theory* posits that a commodity's exchange value is determined by the amount of labour that is required in its production. Clearly, if labour is the only factor of production, the labour demand and labour cost approaches are identical.

Recognising that there is no natural numéraire, Ricardo proposes gold as a standard for measuring exchange value and relative prices. While use value is a necessary but not sufficient condition for a commodity's exchange value, he identified scarcity and the quantity of labour embedded in a commodity as the main determinants of exchange value. According to this view, the exchange value of non-reproducible commodities is solely determined by demand, whereas factor inputs, in particular labour, drive the exchange value of reproducible commodities.

Much of Marx's analysis operates within the classical labour cost theory whereby the exchange value of a good is determined by the amount of labour used in its production. The exchange value of a good is therefore determined by the amount of "socially necessary labour" used in its production. In contrast to classical theory, however, Marx defines labour as a commodity

that is *permanently reproduced*, rather than simply a factor of production. Because it generally takes less than one unit of labour to produce one unit of labour, a unit of labour produces more than its own subsistence requirement. In other words, the labour cost of labour per unit of labour is less than unity which gives rise to a difference between exchange value of labour and its use value. It is this difference, the so-called *surplus value*, from which all profits arise in the Marxian system. While the value of capital is transferred one-for-one in the production process, only labour is capable of producing surplus value – a Marxian notion of value-added. This forms the heart of Marx’s LTV. I will define these terms more formally in section II.3.

As noted above, it is important to bear in mind that classical economists – including Marx – did not establish a link between utility and the determination of exchange value.

II.2 From Values to Prices

Beyond the fact that labour is the cause and determinant of value, another important feature of LTV is the assumption that its own value does not fluctuate. This permits the notion of a natural cost of production which in turn determines the so-called “natural price” of a commodity. While this view does not preclude that the market price of a commodity deviates from its natural price, such deviations are merely viewed as short-run phenomena which do not persist in the long-run. So, contrary to common belief, the LTV does not deny the role of supply and demand influencing prices.

LTV is concerned with the determination of the relative prices of output only and does not explicitly concern itself with the remuneration of the individual factor of production. In classical economics, this is the concern of distribution theory which – broadly speaking – looks at how the exchange value can be decompose into factor shares such as wages, rent (in a Ricardian sense) and profits or interest. In contrast to this clear distinction between valuation and distributional aspects of the mode of production, the prices of factors of production and prices of outputs are determined simultaneously under the neoclassical paradigm. As a result, value theory and distribution theory are merged. As I will argue later, this is to some extent already anticipated by Marx.

II.3 The Marxian Input-Output System

In the following section, I follow [Samuelson \(1983\)](#) and [Blaug \(1997\)](#) when outlining the Marxian version of a simplified input-output system of economic production. For the remainder of this paper, I will rely on the following definitions and identities:

- c = ‘constant capital’, depreciation charges on fixed capital and inputs of raw materials,
- v = ‘variable capital’, workers’ wages,
- s = ‘surplus value’, excess of receipts over $(c + v)$,
- σ = ‘rate of surplus value’, $\sigma \equiv s/v$,
- r = ‘rate of profit’, $r \equiv s/(c + v)$,
- q = ‘organic composition of capital’, $q \equiv c/(c + v)$.

As mentioned above, the assertion that only labour produces surplus value – which in turn drives profits – lies at the centre of Marx’s analysis. Thus it should be possible to determine relative prices from labour values and establish a link between surplus values and profits. However, there are some problems with this line of argument which I will briefly outline below.

Indeed, in Volume I of *Capital*, Marx explained the profits of capital as resulting from surplus value. However, he left open the problem of explaining how capitalists with differing capital-labour ratios, q , can have similar profits. In Volume III of *Capital*, Marx takes up the matter again, and he acknowledges that profit is proportionate to capital rather than labour after all. This “great contradiction” has been at the main focal point for much of the criticism of Marxian economics.

If relative prices are linked to relative labour values and there are no productivity differences, the value of output produced by equal amounts of labour would be the same. With uniform productivity, wage rates are also uniform across industries which implies that the rate of surplus value, σ is the same across all industries. However, different industries clearly have different capital-labour ratios, q , which suggests that profitability, r , should also vary across industries. What is more, a uniform σ and differing qs means that the rate of profit is highest in the most labour-intensive industries – clearly inconsistent with the notion that capitalists substitute capital for labour in search for higher profits. Furthermore, competition between industries leads to an equalisation of profit rates which – despite the fact that the

industry-specific q s are different – cannot be consistent with a uniform σ . This contradiction can be easily visualised as follows:

$$r \equiv \frac{s}{(c + v)} \equiv \frac{\sigma}{(q + 1)} \quad (1)$$

Thus, if the rate of profit, r , is uniform across industries, both σ and q must either be equal or vary inversely. But since q is known to be different across industries, σ must also differ which directly contradicts Marx's own assumption that profits depend only on labour values, i.e. that σ is constant. As relative prices can therefore not correspond to relative labour values, Marx's LTV must be salvaged in some other way, namely by transforming values into prices while maintaining equal rates of surplus and profit across industries. The solution to the great contradiction is the so-called *transformation problem* which is described in the next section.

III The Transformation Problem

In light of the previous discussion, [Desai \(1988\)](#) highlights that the crucial question at the heart of the transformation problem is not so much one of deriving prices from values but one of providing a theory of profits as arising from surplus values.

In Vol. III of *Capital*, Marx himself provided a first solution to the transformation problem which is summarised in this section. The first step in Marx's solution is to define the total value produced by department i . Recalling the definitions of variable in the previous section, this can be written as

$$a_i = c_i + v_i + s_i = c_i + (1 + \sigma)v_i = \frac{(1 + \sigma(1 - q_i))}{(1 - q_i)}v_i. \quad (2)$$

The next step is to express the price of output in terms of the industry-wide profit rate. This can be written as

$$p_i = (1 + r)(c_i + v_i) = \frac{(1 + \sigma(1 - q_0))}{(1 - q_i)}v_i, \quad (3)$$

where q_0 is the industry-wide organic composition of capital. A direct comparison of equations 2 and 3 gives the basic result of Marx's solution to the transformation problem

$$\frac{p_i}{a_i} = \frac{(1 + \sigma(1 - q_0))}{(1 + \sigma(1 - q_i))} \geq 1 \text{ as } q_i \geq q_0. \quad (4)$$

In other words, the deviation of prices from their corresponding values depends on department i 's capital-labour ratio in comparison to the industry average. If department i is more capital intensive than the average, prices of production will exceed the values of the goods and vice versa.

Table 1: **The Great Contradiction**^a

<i>Industry</i>	<i>Capital</i>	<i>c</i>	<i>v</i>	<i>Cost price</i>	<i>s at</i> $\sigma = 1$	<i>Value</i>	<i>Profit</i>	<i>Price of prod.</i>	<i>Price > value</i>
Dept. I	80C + 20V	50	20	70	20	90	20%	92	+2
Dept. II	70C + 30V	51	30	81	30	111	30%	103	-8
Dept. III	60C + 40V	51	40	91	40	131	40%	113	-18
Dept. IV	85C + 15V	40	15	55	15	70	15%	77	+7
Dept. V	95C + 5V	10	5	15	5	20	5%	37	+17
Σ	390C + 110V	202	110	312	110	422	22%	422	0

^a Following Blaug's notation, capital letters are used for *stocks* and lowercase letters for *flows*. Thus, for example, in the case of Marx's 'constant capital', c is defined as the sum of the depreciation charges on fixed capital and inputs of raw materials whereas C is the value of the stock of physical equipment, machinery and inventory of raw materials. *Source*: Marx's original example reproduced from Blaug (1997, p.219).

Table 1 provides a simple illustration of Marx's solution to the transformation problem by considering an economy with five industries that each have the same amount of capital invested and – by construction – enjoy the same σ . Two elements of this example are particularly worth recalling as the basic result presented in equation 4. First, for none of the industries does the value correspond to the price of production, although at the aggregate level of the economy the sum of the price-value deviations is zero. Second, the direction of the deviation of the price of production from value depends on the organic composition of capital.

Table 2 provides a further illustration of this phenomenon in the context of a simple three-industry case. Here, the organic composition of capital

Table 2: **The Transformation Problem – I^a**

<i>Industry</i>	<i>Capital</i>	<i>Cost price</i>	<i>s at $\sigma = 1$</i>	<i>Value</i>	<i>Profit $r = 0.33$</i>	<i>Price of prod.</i>	<i>Price > value</i>	<i>q_i</i>
Dept. I	250C + 75V	325	75	400	108.3	433.3	+33.3	3.3
Dept. II	100C + 50V	150	50	200	50.0	200.0	0	2.0
Dept. III	50C + 75V	125	75	200	41.6	166.6	-33.3	0.7
Σ	400C + 200V	600	200	800	200	800	0	

^a The “organic composition of capital”, q , is not explicitly defined by Marx, but most commonly assumed to be $\frac{c}{(c+v)}$. However, rather than the ratio of two flows, Blaug argues that Marx must have been interested in the ratio of machine cost to labour cost. Therefore, q is defined here as $\frac{C}{v}$. *Source*: Marx’s original example reproduced from [Blaug \(1997, p.220\)](#).

for department II is deliberately constructed in order to correspond to the industry average ($q_2 = q_0$) which is why its value is identical to the price of production. Department I has an organic composition of capital in excess of the industry average, whereas the opposite is the case for department III ($q_1 > q_0$ and $q_3 < q_0$). Thus with a given σ and industry-wide profit equalisation, for example, department III can only earn the average rate of profit which means that the price of production is less than the value. As is implied in tables 1 and 2, therefore, the inter-industry equality in the rate of profit causes surplus to be redistributed from labour-intensive industries to capital-intensive industries.

III.1 Solving the Transformation Problem

At first sight, Marx’s solution seems to provide a satisfactory theory of profits arising from surplus value. Indeed, prior to the publication of Vol. III, Engels famously issued the challenge to solve the transformation problem, simultaneously announcing that the forthcoming Vol. III would settle all controversy. Yet, he was (partially) wrong as overcoming this paradox still captures economists attention even 125 years after it was first formulated by Marx. The following section provides an overview of some of the main solutions to the transformation problem.

In Marx’s description of the transformation problem, industries are not related with each other. However, any general solution of the transforma-

tion problem should be able to link all values – both input and outputs – to the corresponding relative prices. In modern terminology, the transformation problem is therefore a *general equilibrium proposition* about how various sectors interact and how the rate of profit is equalised through price of production-value differences as illustrated in the previous section.

III.2 Böhm-Bawerk and Bortkiewicz

Perhaps one of the most famous criticisms of the apparent inconsistency in Marx’s LTV is due to [von Böhm-Bawerk \(1898\)](#) who maintains that Marx does not resolve the issue logically. The fact that rates of profit rather than surplus value tend towards equality across industries implies that that commodities will sell at their cost of production rather than their labour value. Thus, the process of transforming values into prices necessarily determines the labour theory of value in its entirety.

“I cannot help myself; I see here no explanation and reconciliation of a contradiction, but the bare contradiction itself. Marx’s third volume contradicts the first. The theory of the average rate of profit and of the prices of production cannot be reconciled with the theory of value. This is the impression which must, I believe, be received by every logical thinker . . . And even a man who is so close to the Marxian system as Werner Sombart, says that a “general head-shaking” best represents the probable effect produced on most readers by the third volume.”²

Böhm-Bawerk alleges that Marx’s theory of prices of production stands in direct contradiction with the theory that exchange values are determined by the quantity of labour “socially necessary” to produce a commodity. In other words, equal values correspond to equal prices only in industries for which the organic composition of capital is equal to the industry average, ie. $q_i = q_0$.

Furthermore, going beyond Marx’s original analysis of “simple reproduction” where there is no economic interaction between the three departments, the transformation problem also needs to be solved for “expanded reproduction”. The mathematical solution to this generalised version of the transformation problem is due to [von Bortkiewicz \(1907\)](#) and was later elaborated

²[von Böhm-Bawerk \(1898\)](#), chapter III, “The Question of the Contradiction”.

by Winternitz (1948).³ This solution is illustrated below. For this purpose, Department I is now assumed to produce capital goods which are used as intermediate goods by departments II and III. Department II produces wage goods consumed by workers and department III supplies luxury goods for the capitalist class.

Table 3: **Transformation Problem – II^a**

	<i>Values</i>	<i>Prices of production</i>
Dept. I	$c_1 + v_1 + s_1 = a_1$	$c_1 p_1 + v_1 p_2 + \pi_1 p_3 = a_1 p_1$
Dept. II	$c_2 + v_2 + s_2 = a_2$	$c_2 p_1 + v_2 p_2 + \pi_2 p_3 = a_2 p_2$
Dept. III	$c_3 + v_3 + s_3 = a_3$	$c_3 p_1 + v_3 p_2 + \pi_3 p_3 = a_3 p_3$
Σ	$a_1 + a_2 + a_3 = \Sigma a$	$c p_1 + v p_2 + \pi p_3 = \Sigma a p$

^a Source: Modified from Winternitz (1948) and Blaug (1997).

Table 3 illustrates how this mode of production can be represented by two systems of equations that relate values (c_i, v_i, s_i) to total output (a_i) and prices of production (p_i). Solving the transformation problem can thus be viewed as finding a unique solution relative prices solution.

Since profits are defined as $\pi_i = r(c_i p_1 + v_i p_2)$ and profitability is assumed to be identical in department I and II, the rate of profit can be written as $r = \pi_i / (c_i p_1 + v_i p_2)$.⁴ This now permits to express the value of total output in terms of the rate of profit as $a_i p_i = (1 + r)(c_i p_1 + v_i p_2)$. Rearranging terms, this can then be re-written as

$$1 + r = \frac{a_i p_i}{c_i p_1 + v_i p_2} \text{ for } i = 1, 2. \quad (5)$$

In his original solution, Winternitz (1948) demonstrates that equation 5 can be written as the following a quadratic equation by re-arranging and setting $m \equiv \frac{p_1}{p_2}$. Thus

³Economic historians do not uniformly share the same assessment of the relative merits of these solutions. While Desai (1988, p.297), for example, lauds Bortkiewicz's "less turgid and elegant solution", May (1948, p.596) refers to the same efforts as "an artificial confusion [with] pseudo-mathematical mystifications" that bear little relation to the basic problem posed by Marx.

⁴Department III produces luxury consumption goods and has no influence on average profits by construction.

$$m^2(a_1c_1) + m(a_1v_2 - a_2c_1) - (a_2v_1) = 0, \quad (6)$$

which has the solution

$$m = \frac{(a_1v_2 - a_2c_1) + \sqrt{(a_2c_1 - a_1v_2)^2 + (4a_1a_2v_1c_2)}}{2a_1c_2}. \quad (7)$$

Ignoring negative solutions, m is now given and the average rate of profit is determined as

$$r = \frac{a_1m}{c_1m + v_1} - 1. \quad (8)$$

It is important to note that this solution is somewhat restrictive for two reasons. First, the rate of profit in department III and its organic composition of capital have no influence on the average rate of profit. In other words, this three-industry model assumes that the final use of a commodity is predetermined by the department in which it is manufactured. Second, because the transformation problem has been solved in terms of relative prices (recalling that $m \equiv \frac{p_1}{p_2}$), additional aggregate characteristics are required to determine absolute prices.

Winternitz suggests to choose Marx's proposition that the "sum of prices is equal to the sum of values" (i.e. the labour theory of prices). However, [Blaug \(1997\)](#) notes that there is a second invariance condition that has equal theoretical merit, namely that "the total surplus in value terms is equal to profits in price terms" (i.e. the labour theory of profits).

This now raises the complication that with two invariance conditions and three department equations as displayed in table 3, the transformation problem is *overdetermined* since there are five equations but only four unknowns (r, p_1, p_2 and p_3). This means that a general solution of the transformation problem is only possible by retaining one of the invariance conditions, but not both. In other words, it is only possible under very restrictive assumptions to solve Marx's original challenge of applying the LVT to both relative prices and profits.

III.3 Beyond Mathematics

However, beyond these mathematical complications outlined in the previous section, the solution of the transformation problem shows that if value and price are defined according to Marx, there exists a simple transformation connecting the two. The transformation is independent of any equilibrium condition. Thus the transformation problem – in the strictly technical sense of linking value and price of production – is seen to be relatively straightforward mathematically.

However, even if it can be accepted that the transformation problem can be tackled algebraically, there is a number of other issues that remain unresolved and are keeping economists occupied with this issue. A number of these qualitative aspects of solutions to the transformation problem is described in the next section. Many subtle issues regarding the transformation problem reach far beyond its mathematical solvability.

May (1948) points out that the real problem in the context of Marxian economics is not the difficulty of relating values and prices of production, but relating these concepts to *actual prices* which are not expressed in terms of labour time in the production process. This is then one of the weaknesses of the mathematical solution of the transformation problem elaborated by Winternitz (1948) who conflates prices of production with prices at which exchange takes place. Indeed, May highlights that – in Marx’s original sense – the price of production is a form of value and that price may not only deviate from price of production, but a commodity may also have a price without having a price of production.

To complicate things further, s and v are not only non-observable, but also non-behavioural variables. This makes it particularly problematic to formulate a priori whether σ , the rate of surplus value, is equal across industries or not. The transformation problem stands traditional economic reasoning and reality on its head in so far as observable prices have to be transformed into unobservable values and not the other way round.

IV Two Interpretations

By the mid-20th century numerical solutions to the transformation problem were sufficiently generalised and mostly universally accepted. Increasing computing power in the 1960s also made it possible to devise real-world

applications to the transformation problem by deriving values and prices of production from actual real prices. At the height of the cold war in the 1960s and 1970s, this was particularly popular practice in COMECON countries. The most ambitious such project took place in Yugoslavia where a group of economists transformed current prices into prices of production on the basis of a 28-sector input-output model (Bajt, 1970, p.371).

Even after these practical applications, there still seemed to remain sufficient substance for debate among economists. Meek (1956) argues that any mathematical or logical solution of the transformation problem would only fill part of the gap in Marx’s analysis. To fill the rest of it, one must turn to economic history rather than mathematics. Specifically, he suggested that the “derivation of prices from values” must be regarded as a historical as well as a logical process. As such, the transformation problem can be viewed as a historic description of the evolution of the capitalist laws of motion as opposed to the mode of production in a pre-capitalist society. This aspect of the transformation problem is still actively debated in Marxian economics today and is briefly revisited in section IV.2.

In the latter half of the last century, the treatment of the transformation problem was increasingly conducted in two diametrically opposed intellectual camps – mainstream economics on the one hand, and radical, heterodox economists on the other hand. This section looks at some of the similarities and differences of how the transformation problem has been treated, why it was put to rest and then re-suscitated, and – most recently – how it has been re-invented.

IV.1 Transforming the Mainstream?

The centenary of the publication of Marx’s *Capital* saw a revival of the debate about the transformation problem in mainstream economics. While much of this analysis acknowledges the historical importance of Marx’s LTV, there soon was consensus that – with the propositions of marginalist revolution now a mainstay of economic reasoning – the relative prices of goods really do change as demand changes, even when their socially-necessary labour contents do not change. Indeed, in a early reprise on the transformation problem Samuelson (1967) (rhetorically) concedes that

“[...] if labour-theory-of-value reasoning, as applied to an impeccable model of equal factor intensities, turned up new light on exploitation

in an existing system or if it turned up new light on the laws of development of capitalism, it would be an invaluable tool even though not defensible as a general theory of markets. ”⁵

Perhaps not surprisingly, however, he concludes that “Marxian economics is powerless to explain the 1937–1967 developments of European and American economics”.⁶ Later, Samuelson (1971, 1974) asserts that the transformation problem was in fact a “non-problem” that could be eliminated mathematically. In his view, the transformation problem was then not the Achilles heel of Marxian economics but simply a redundant appendix.⁷ Samuelson’s focus on the irrelevance of the transformation problem stems from the practical reality that prices and profits could be derived directly from input-output data without the detour of computing values.

At the same time, however, other mainstream economists argued differently and highlighted that the apparent inconsistency between Marx’s two theories of value is quite deliberate. While Vol. I might give the impression that the labour values of commodities were intended to explain their exchange values (relative prices), Niehans (1990), for example, claims that it would be wrong to conclude that Marx was proposing a labour theory of exchange value. Indeed, he highlights that precisely this difference between the two sets of values is one of Marx’s major contributions. As such, criticising this deliberate difference between the two (labour account, exchange account) is besides the point.

Thus the question arises whether Marx was simply wrong – or perhaps inconsistent – or why would he deliberately mislead? Baumol (1974a,b) is convinced it is the latter, yet his answer is rather surprising. He maintains that Marx did not intend his transformation analysis to show how prices can be deduced from labour values. Rather, because the two sets of values are derived independently and because they differ in a significant and systematic manner, Marx had set deliberate traps for the express purpose of goading “vulgar economists”.

In this view, the relevance of the transformation problem for mainstream economics stems from the general equilibrium insights in which surplus values were redistributed through the economic system via the pricing mechanism

⁵Samuelson (1967, p.620).

⁶Samuelson (1967, p.623).

⁷See Bronfenbrenner (1973) for a concise discussion and overview of Samuelson’s “vituperative feuding with Marx’s ghost”.

to equalise profit rates. Indeed, Baumol argues that this central insight of Marx' LTV had been lost in the increasingly mathematical treatment of the transformation problem, starting with Böhm-Bawerk right up to Samuelson.

“My contention is that Marx' interest in the transformation analysis as a sequel to his value theory was not a matter of pricing. Rather it sought to describe how *non* wage incomes are *produced* and then how this aggregate is *redistributed* [...] the substance of Marx' analysis can be summarised in a simple parable, in which the economy is described as an aggregation of industries each of which contributes to a storehouse containing total surplus value [...] if we use labour units to measure these quantities, each industry's contribution is proportionate to the quantity of labour it uses.”⁸

IV.2 The “New Solution”

In contemporary Marxist economics, the meaning and significance of the transformation problem goes well beyond a simple mapping of values onto prices and linking surplus value to profits. Broadly speaking, there are two main schools of thought within heterodox economics, each with a very different take on the nature and significance of Marx's LTV and the transformation problem.⁹

On the one hand, there is the so-called *rationalist interpretation* that relies on the more philosophical distinction between essence and appearance. According to this interpretation, the essence of something is never given immediately in its empirical form. Abstract labour, therefore, is the substance whose empirical form is the price. In this approach the issue is not whether one can find a general mathematical solution for transforming abstract labour and value into prices. Indeed, values are only empirically observable as prices.

In contrast to this view, there is the *empirical interpretation* which – accepting that values are the only cause of prices – views the transformation problem as a mathematical specification to account for the divergence of observed prices from values. Thus in its mechanics, though not the assumptions, this school of thought is close to the treatment of the transformation problem within mainstream economics. Out of this school of thought, a new branch of economic literature emerged in the 1980s that deals extensively with more

⁸Baumol (1974b, p.53), italics in the original.

⁹See Hunt (1989) for a comprehensive overview of this literature.

mainstream criticism of the transformation problem of mathematical and logical inconsistency that were discussed in section III.1.

The so-called “new solution” (NS) of the transformation problem was first independently put forth by Duménil (1984a,b) and Foley (1984). This new approach rests on treating two aspects differently than in the traditional formulation. First, net value is used instead of gross value to avoid double counting. Second, the division of new value into variable capital and surplus value is determined in terms of money wages paid to workers rather than workers’ consumption goods as is the case in the traditional approach.¹⁰ Defining a direct role for money wages in the distribution of surplus value – rather than hypothetical workers’ consumption bundles – is perhaps the most significant contribution to the transformation problem by the NS literature.

Yet, while some hail the NS as “a very important advance in Marxian scholarship”, others are much more cautious and warn that the NS is premised on theoretical modifications that “cannot be supported easily by textual evidence from Marx’s work”.¹¹

V Outlook

Transformation problem – deriving prices from values and providing a theory of profits as arising from surplus values – and possible solutions to the problem have received widespread attention across a wide range of theoretical and empirical economics. Unlike only few other issues in the dismal science, it still elicits interest and has the capacity to polarise economists – both in the mainstream and at the fringe. Undeniably, the transformation problem continues to fascinate and antagonise as much today as it did when it was first formulated by Marx over 125 years ago. To many, Eugen von Böhm-Bahwerk’s somewhat caustic assessment may still capture this sentiment best:

“I consider it one of the most striking tributes which could have been paid to Marx as a thinker that this challenge [*the transformation problem*] was taken up by so many persons, and in circles so much wider than the one to which it was chiefly directed . . . even economists who

¹⁰See Campbell (1997, 2002) for a comprehensive review of the literature on the “new solution” to the transformation problem.

¹¹See eg. Moseley (2000, p.312) and a more critical assessment of the NS by Rieu (2006, p.259).

would probably have been called by Marx “vulgar economists”, vied with each other in the attempt to penetrate into the probable nexus of Marx’s lines of thought, which were still shrouded in mystery. There grew up between 1885, the year when the second volume of Marx’s *Capital* appeared, and 1894 when the third volume came out, a regular prize essay competition on the “average rate of profit”, and its relation to the “law of value”. According to the view of Friedrich Engels - now, like Marx, no longer living - as stated in his criticism of these prize essays in the preface to the third volume, no one succeeded in carrying off the prize.”¹²

While there may be disagreement among those who claim that they have at least partially succeeded in its solution, there are several aspects of the transformation problem that remain unresolved. As it has been dealt with to date, the transformation problem only presents itself as a static derivation of prices from values and profits from surplus values. It has neither been carried out in dynamic terms yet, nor has it been extended to include monopolistic competition. Furthermore, it has no role for money and does not deal with uncertainty.

Indeed, it can be argued that the transformation problem has been solved by Marx himself and little conceptual progress has been made in the over hundred years since then. While much of the work and effort that has been directed towards the transformation problem has helped to a much clearer formulation and solution of the problem itself, the key aspects of dynamics, uncertainty, and money remain untackled. Indeed a large field for future research that certainly warrants attention from all throughout the dismal science.

¹²[von Böhm-Bawerk \(1898\)](#).

References

- BAJT, A. (1970): "A Post Mortem Note on the 'Transformation Problem'," *Soviet Studies*, 21(3), 371–374.
- BAUMOL, W. J. (1974a): "The Fundamental Marxian Theorem: A Reply to Samuelson: Comment," *Journal of Economic Literature*, 12(1), 74–75.
- (1974b): "The Transformation of Values: What Marx "Really" Meant (An Interpretation)," *Journal of Economic Literature*, 12(1), 51–62.
- BLAUG, M. (1997): *Economic Theory in Retrospect*. Cambridge University Press, Cambridge, U.K., fifth edn.
- BRONFENBRENNER, M. (1973): "Samuelson, Marx, and Their Latest Critics," *Journal of Economic Literature*, 11(1), 58–63.
- CAMPBELL, A. (1997): "The Transformation Problem: A Simple Presentation of the "New Solution"," *Review of Radical Political Economics*, 29(3), 59–69.
- (2002): "The Nature of Surplus Value in the "New Solution"," *Review of Radical Political Economics*, 34(1), 69–72.
- DESAI, M. (1988): "The Transformation Problem," *Journal of Economic Surveys*, 2(4), 295–333.
- DUMÉNIL, G. (1984a): "Beyond the Transformation Riddle: A Labor Theory of Value," *Science & Society*, 47, 427–450.
- (1984b): "The So-called "Transformation Problem" Revisited: A Brief Comment," *Journal of Economic Theory*, 33, 340–348.
- FOLEY, D. K. (1984): "The Value of Money, the Value of Labour Power and the Marxian Transformation Problem," *Review of Radical Political Economics*, 14(2), 37–47.
- HEILBRONER, R. L. (1999): *The Worldly Philosophers: The Lives, Times And Ideas Of The Great Economic Thinkers*. Touchstone, seventh edn.

- HEILBRONER, R. L., AND W. S. MILBERG (1995): *The Crisis of Vision in Modern Economic Thought*. Cambridge University Press, Cambridge, UK, first edn.
- HUNT, E. K. (1989): "The Meaning And Significance Of The Transformation Problem: Two Contrasting Approaches," *Atlantic Economic Journal*, 17(4), 47–54.
- MAY, K. (1948): "Value and Price of Production: A Note on Winternitz' Solution," *Economic Journal*, 58(232), 596–599.
- MEEK, R. L. (1956): "Some Notes on the "Transformation Problem"," *Economic Journal*, 66(261), 94–107.
- MOSELEY, F. (2000): "The "New Solution" to the Transformation Problem: A Sympatheic Critique," *Review of Radical Political Economics*, 32(2), 282–316.
- NIEHANS, J. (1990): *A History of Economic Theory: Classic Contributions 1720–1980*. Johns Hopkins University Press, Baltimore and London.
- RIEU, D.-M. (2006): "A Reexamination of the Quantitative Issues in the New Interpretation," *Review of Radical Political Economics*, 38(2), 258–271.
- RIMA, I. H. (2001): *Development of Economic Analysis*. Routledge, London, sixth edn.
- ROBBINS, L. (1998): *A History of Economic Thought: The LSE Lectures*. Princeton University Press, Princeton and Oxford.
- SAMUELSON, P. A. (1967): "Das Kapital: A Centenary Appreciation – Marxian Economics as Economics," *American Economic Review*, 57(2), 616–623.
- (1971): "Understanding the Marxian Notion of Exploitation: A Summary of the So-Called Transformation Problem Between Marxian Values and Competitive Prices," *Journal of Economic Literature*, 9(2), 399–431.
- (1974): "Insight and Detour in the Theory of Exploitation: A Reply to Baumol," *Journal of Economic Literature*, 12(1), 62–70.

- (1983): *Foundations of Economic Analysis*, Harvard Economic Studies. Harvard University Press, Cambridge, MA, enlarged edn.
- SKOUSEN, M. (2007): *The Big Three in Economics: Adam Smith, Karl Marx and John Maynard Keynes*. M. E. Sharpe, Armonk, NY.
- VON BÖHM-BAWERK, E. (1898): *Karl Marx and the Close of His System*. T.F. Unwin, London.
- VON BORTKIEWICZ, L. J. (1907): “Wertrechnung und Preisrechnung im Marxschen System,” *Archiv für Sozialwissenschaft und Sozialpolitik*, 34(3), 370–385.
- WINTERNITZ, J. (1948): “Values and Prices: A Solution of the So-called Transformation Problem,” *Economic Journal*, 58(230), 276–280.